

§ 1. Integración inmediata

1°. REGLAS PRINCIPALES PARA LA INTEGRACION.

1) Si $F'(x) = f(x)$, entonces

$$\int f(x) dx = F(x) + C,$$

donde C es una constante arbitraria.2) $\int Af(x) dx = A \int f(x) dx$, donde A es una constante.3) $\int [f_1(x) \pm f_2(x)] dx = \int f_1(x) dx \pm \int f_2(x) dx$.4) Si $\int f(x) dx = F(x) + C$ y $u = \varphi(x)$, se tiene,

$$\int f(u) du = F(u) + C.$$

En particular,

$$\int f(ax+b) dx = \frac{1}{a} F(ax+b) + C \quad (a \neq 0).$$

2°. TABLA DE INTEGRALES INMEDIATAS.

I. $\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1.$

II. $\int \frac{dx}{x} = \ln |x| + C.$

III. $\int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C = -\frac{1}{a} \operatorname{arctctg} \frac{x}{a} + C_1 \quad (a \neq 0).$

IV. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C \quad (a \neq 0).$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + C \quad (a \neq 0).$$

V. $\int \frac{dx}{\sqrt{x^2+a}} = \ln |x + \sqrt{x^2+a}| + C \quad (a \neq 0).$

VI. $\int \frac{dx}{\sqrt{a^2-x^2}} = \operatorname{arcsen} \frac{x}{a} + C = -\operatorname{arccos} \frac{x}{a} + C_1 \quad (a > 0).$

VII. $\int a^x dx = \frac{a^x}{\ln a} + C \quad (a > 0); \quad \int e^x dx = e^x + C.$

VIII. $\int \operatorname{sen} x dx = -\cos x + C.$

IX. $\int \cos x dx = \operatorname{sen} x + C.$

X. $\int \frac{dx}{\cos^2 x} = \operatorname{tg} x + C.$

XI. $\int \frac{dx}{\operatorname{sen}^2 x} = -\operatorname{ctg} x + C.$

XII. $\int \frac{dx}{\operatorname{sen} x} = \ln \left| \operatorname{tg} \frac{x}{2} \right| + C = \ln |\operatorname{cosec} x - \operatorname{ctg} x| + C.$

XIII. $\int \frac{dx}{\cos x} = \ln \left| \operatorname{tg} \left(\frac{x}{2} + \frac{\pi}{4} \right) \right| + C = \ln |\operatorname{tg} x + \operatorname{sec} x| + C.$

XIV. $\int \operatorname{sh} x dx = \operatorname{ch} x + C.$

XV. $\int \operatorname{ch} x dx = \operatorname{sh} x + C.$

XVI. $\int \frac{dx}{\operatorname{ch}^2 x} = \operatorname{th} x + C.$

XVII. $\int \frac{dx}{\operatorname{sh}^2 x} = -\operatorname{cth} x + C$

EJEMPLO 1. $\int (ax^2 + bx + c) dx = \int ax^2 dx + \int bx dx + \int c dx = a \int x^2 dx + b \int x dx + c \int dx = a \frac{x^3}{3} + b \frac{x^2}{2} + cx + C.$